Note on recent `arXiv:1801.10246`: Morishima, Futamase, Shimizu 2018
Gravitational field effect on motion of Dirac particle:
$\mu_\mu^{\rm eff} = (1+3\phi/c^2)\,\mu_\mu\;;\quad \phi=-\frac{GM}{r}$ the earth gravitational field
$x \equiv \phi/c^2 = -\frac{GM}{R}/c^2 \approx -7.0\times 10^{-10}$
In an interesting calculation in the post-Newtonian expansion the authors show that the effect drops out at leading order in ϕ/c^2 , hence no shift in a_μ , because it is extracted from measuring frequency ratios.
Afterwards the authors argue that there is an effect related to the change of the coefficient in the $\vec{E}$ field term of Eq. (40): the master equation for extracting a_μ from the Larmor frequency and the applied e.m. fields

$$\vec{\Omega}_a^{\rm eff} = -\frac{e}{m}\Bigg((1+3\,x)\,a_\mu\,\vec{B} - \left[a_\mu - \frac{1}{\gamma^2-1} - x\,\left(4+a_\mu+\frac{3}{\gamma^2-1}\right)\right]\vec{\beta}\times\vec{E} - a_\mu\,\frac{\gamma}{\gamma+1}\,\left(1-x\,(2\gamma-1)\right)\,\left(\vec{\beta}\cdot\vec{B}\right)\,\vec{\beta}\Bigg)$$

For the muon storage ring the last term vanishes as $\vec{\beta}\cdot\vec{B}$ (B-field vertical to plane of motion).
The authors attempt to use Equation (44) defining a modified a_μ
$a_\mu^{\rm mod} = a_\mu - x\,\left(4+a_\mu+\frac{3}{\gamma^2-1}\right)\,,$
which if tuned to magic energy as $a_\mu^{\rm mod}-\frac{1}{\gamma^2-1}=0$ and claim
$a_\mu = a_\mu^{\rm mod} + x\,\left(4+a_\mu+\frac{3}{\gamma^2-1}\right) \simeq a_\mu^{\rm mod} - 28\times 10^{-10}$

to represent the measured anomaly. This would essentially nullify the observed deviation $3.7\sigma \to 0.1\sigma$

In fact a_μ is determined from the leading term of the master equation $\propto \vec{B}$, while the second term $\propto \vec{\beta}\times\vec{E}$ is tuned to zero by choosing the Lorentz factor $\gamma(E)$ to the magic energy $E_{\rm mag}\sim 3.1$ GeV .
Another fact: what is measured?
a_μ measured via a ratio of frequencies (measurement of a_μ and B)
$B = \frac{\hbar\omega_p}{2\mu_p}\,,\ \omega_a = \frac{e a_\mu}{m_\mu c}\,B\,,\ \mu_\mu = (1+a_\mu)\,\frac{e\hbar}{2m_\mu c} \ \Leftrightarrow\ \mu_\mu = (1+a_\mu)\,\frac{\hbar}{2}\,\frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu}+1\right)\,\frac{\omega_a}{\omega_p}\,\mu_p \Leftrightarrow$

$\boxed{a_\mu = \frac{\mathcal{R}}{\lambda - \mathcal{R}}}$ in terms of 3 frequency measurables

- ❒ $\tilde{\omega}_p = (e/m_\mu)\langle B \rangle$ free proton NMR frequency
- ❒ $\mathcal{R} = \omega_a / \tilde{\omega}_p$ Larmor precession from E-821
- ❒ $\lambda = \omega_L / \tilde{\omega}_p = \mu_\mu / \mu_p$ from hyperfine splitting of muonium

value used by E-821 3.18334539(10)

new value 3.183345137(85) Mohr et al RMP 80 (2008) 633

It is easy to check that the gravity correction $(1 + 3\,x)$ in the leading term of the master equation drops out in all ratios as correctly demonstrated in the paper.

Evidently a precise energy adjustment nullifying the second term depends on the earth gravity correction (in red). Note, what has to be tuned is the energy factor γ and not a_μ and the question is what a possible false tune amounts to:

magic gamma in earth field	=	29.303505565136842
magic gamma no gravity	=	29.303470378324022
$\delta\gamma_{\mathrm{mag}}$	=	3.7177814463850568E-003
E_{mag} in earth field	=	3.0961607651640625 GeV
E_{mag} no gravity	=	3.0961570473826159 GeV
$\delta E/E\mathrm{mag}$	=	1.2007728902307946E-006

Thus the earth gravity requires an fractional energy shift 1.2×10^{-6} relative to the gravity free case, far below the beam energy resolution of the Brookhaven/Fermilab experiment, which has been 0.5% and for which the experiment is corrected as [equivalent to changing the effective magnetic field]

$$\frac{\Delta\omega_a}{\omega_a} = C_E \simeq -2\frac{\beta E_r}{B_0}\left(\frac{\Delta p}{p_m}\right) . \tag{1}$$

[$C_E \approx 0.5$ ppm for BNL].

Using actual E- and B-field values:

For BNL/Fermilab E-field is $E_r \simeq 30\mathrm{kV/5cm} \sim 6 \times 10^5$ V/m , the B-field $B_0 \simeq 1.5$ Tesla $[= \mathrm{Vsec/m}^2]$, muon velocity $\beta = v/c \simeq 1$, in MKS units $p = \beta\gamma m/c$ i.e. $\beta \to \beta/c \simeq 1/3 \times 10^8$ sec/m such that $\beta/cE_r/B_0$ about 1300 ppm which is multiplying $\delta E/E\mathrm{mag}$ given above, is yielding about 3 ppb from the earth gravity correction. The experimental accuracy was 560 ppb for BNL and will be 140 ppb at Fermilab. Therefore the gravity shift is much smaller than the experimental precision.

More details may be found in my book Chap. 6.3.1 see specifically p. 584 and references there.

But it is an important consideration anyway as
$\tau_\mu = 2.197 \times 10^{-6}$ sec ; $g = 9.81$ m/sec^2 ; height of fall $h = \frac{1}{2} g t^2 \simeq 23.68 \times 10^{-12}$ m with time dilatation at magic momentum $\gamma \simeq 29.3$ such that $h \approx 2.03 \times 10^{-8}$ m
which however is compensated by the focusing field adjustment I guess in case the gravitational displacement should be relevant in this experimental setup at all.
What experiments do I guess is
1) calculate $\gamma_{\rm mag}$ assuming some approximately known value for a_μ
2) measure $a_\mu^{\rm exp}$ and use it to recalculate $\gamma_{\rm mag}$
3) repeat until convergence.
Here it is true that the calculated $\gamma_{\rm mag}$ depends on whether the gravitational correction is included or not. But the shift is far within the experimental resolution.
Only our experimental colleagues can definitely answer how they precisely adjust the magic energy and how this affects their result.
Provided p is properly tuned to the magic value a_μ is independent of the electric field E , and of course J-PARC will measure the same a_μ at $E = 0$. It is amazing that the fact that the shift by $3\phi/c^2 \sim -2.1 \times 10^{-9}$ in the leading term of the master equation actually drops out form the measurement because of the equivalence principle of gravity i.e. the effect is the same for the muon and the proton and drops out in the ratio. This is the important message of the paper! Otherwise indeed we would have a problem. That's why I think the paper has been addressing an important issue.

Summary:
1) The earth gravitational field potentially could reduce the effective magnetic moment of a particle (electron, muon, proton ...) by a correction factor $(1 + 3\phi/c^2)$ i.e. by a fractional correction $3\phi/c^2 \simeq -2.1 \times 10^{-9}$, which is of the size of the 3.7 σ deviation observed in case of the muon.
2) the same effect would be seen in case of the electron, where we know that no deviation is observed
3) We also note that this magnetic moment would depend on the size of the gravitational field at the location where it is measured, i.e. it would not be a property attributed to the particle itself only. A proper definition would require to correct for the gravitational effect (as we do in many cases with electromagnetic radiative corrections)

4) In fact the way the anomalous magnetic moments of the electron and muon are measured are automatically devoid of the gravitational correction if measured in a homogeneous magnetic field at vanishing electric field. This is a consequence of the equivalence principle of gravity. The measuring definition is in terms of ratio of frequencies as given above.
5) The effect of the gravitational field in principle affect the magic energy tuning, which however is very small relative to the precision which can be reached. Standard results ignoring gravity remain valid to high accuracy within uncertainties.

F. Jegerlehner, Wildau, February 2-6, 2018