

Heavy Flavour Benchmarks of ILD

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Abstract

An overview of the performance of the ILD detector in its version Large and Small as relevant for the IDR is given

1. Introduction

Heavy quarks may be messengers of new physics of primary importance. High precision e^+e^- collisions with polarised beams around the TeV scale are ideally suited to detect new physics effects [1, 2]. The detection of the onset of new physics require however a superb detector performance in terms of flavor tagging including the event by event determination of the charge of the final state jets. The charge determination happens mainly by a combination of the determination of the summed charge of tracks pointing to a secondary vertex or by the identification of the charge of a final state Kaon. This in turn requires a successful particle identification by the detector. Therefore processes with heavy quark final states, i.e. $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow t\bar{t}$ are highly relevant for the benchmarking of the detector performance. In short one can test the following detector capacities.

- Track finding efficiency
- Stringent test of (secondary) vertexing
- Particle ID
- In case of $e^+e^- \rightarrow t\bar{t}$ leptonic and semi-leptonic decays of the top-quark pair provide an important additional handle for the accurate measurement of the final state. The analysis presented in this note focuses on the semi-leptonic decay mode of the top-quark pair.

The analyses presented in this note start out from the PhD thesis of Sviatoslav Bilokin that are based on the DBD samples and software versions [3]. This work has in part been published in Ref [2]. The analyses are ported to the large, IDR-L, and small, IDR-S, detector models, respectively, and are carried out with the software version that is relevant for the IDR of ILD. For the process $e^+e^- \rightarrow b\bar{b}$ an analysis at $\sqrt{s} = 500$ GeV is presented instead of $\sqrt{s} = 250$ GeV as in Ref. [2]. The results here benefit from a refined analysis strategy for the ILD paper that is under review in ILD.

2. Methods, tools and Monte Carlo samples

For the event reconstruction we use the `ILCSOFT` version `v02-00-02` We use the following methods

- ‘Core tools’
 - We use the `ValenciaVertex` jet algorithm implemented in `LCFIPlus` that provides the rejection of $\gamma\gamma$ background. In this algorithm the distance between two objects is calculated as

$$d_{ij} = 2 \min(E_i^{2\beta}, E_j^{2\beta})(1 - \cos \theta_{ij})/R^2 \quad (1)$$

34 The distance of a particle i to the beam is calculated according to.

$$d_{iB} = E^{2\beta} \sin^{2\gamma} \theta_{iB} \quad (2)$$

35 The jet algorithm is run with the following settings: $\alpha = \beta = \gamma = 1$, $R = 1.4$

- 36 – We use the `LeptonFinder` in case of semi-leptonic $e^+e^- \rightarrow t\bar{t}$ events
 37 – For the vertex finding we use the `LCFIPlus` in a private version maintained by Ryo. This
 38 version will soon be implemented into the official ILCsoft package.

39 • Tools developed for the study

40 – The `VertexRestorer` Processor identifies tracks that have not been associated to secondary
 41 vertices from B-Meson decays but belongs to this decay according to the Monte Carlo
 42 Truth information. It then recovers the ‘lost’ tracks by means of the impact parameters d_0
 43 (transversal) and z_0 (longitudinal). In this present note the recovery uses only the impact
 44 parameter d_0 since the algorithms needs to be adapted for the vertex smearing present in
 45 the simulation for the IDR.

46 – The `ParticleTagger` Processor identifies the Kaons by means of the dE/dx measured
 47 in the TPC of ILD. It selects a strip in the dE/dx -momentum plane with a high kaon
 48 concentration. The efficiency and the purity of the Kaon selection vary as a function of the
 49 width of this strip.

50 – The `QQbarAnalysis` Processor calculates the jet charge and the polar angle of the bottom
 51 and top quark pair, respectively. It contains separate methods for the bottom and top quark
 52 pair analysis.

53 – The `TrashRecoProcessor` enables comparisons between reconstructed and generated quan-
 54 tities.

55 – The described tools are available under <https://github.com/qqbaranalysis>. This reposi-
 56 tory contains also a set of macros necessary for the final steps of the analysis.

57 • Quark charge measurement and corrections for miscalculations

58 – Probabilities on double charge measurements for $t\bar{t}$ and $b\bar{b}$ has been examined.

59 – Calculations scheme is shown below.

$$\left. \begin{aligned} N_{acc} &= Np^2 + Nq^2 \\ N_{rej} &= 2Npq \\ 1 &= p + q \end{aligned} \right\} N_{corr} = N_{acc} \cdot \frac{p^2}{p^2 + q^2} \quad (3)$$

60 – where N is total number of events, N_{acc} and N_{rej} are number of events that were accepted
 61 and rejected, respectively. The p and q values represent the probabilities for a correct or
 62 an incorrect reconstruction of the b quark charge, respectively. Solving this equation allows
 63 to correct for migrations caused by imperfect reconstruction or $B^0 - \bar{B}^0$ oscillations in the
 64 resulting polar angle spectrum.

65 – the correction has been applied to the $b\bar{b}$ studies but not (yet) for $t\bar{t}$. For the latter the
 66 selection scheme in $t\bar{t}$ is more involved rendering its application less straight forward since
 67 e.g. the b -quarks are not necessarily back-to-back.

2.1. Monte Carlo samples and Event processing

For this benchmark study only processes with *left-handed* electron beam polarisation and *right-handed* positron beam polarisation, i.e. $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \mp 1$, have been studied so far. The final states resulting from this configuration are more demanding for the detector performance in terms of the control of migration effects. In both cases the samples available for small and large detectors are available and comparisons will be presented where appropriate.

In case of the top-quark study we have analysed samples for the beam polarisations $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \pm 1, \mp 1$. In case of the bottom quark study we have studied only the configuration $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \mp 1$ since only a restricted simulated sample was available and this study was initially just considered as a supporting study for the top-quark analysis.

More precisely the results presented in this note are based on the following samples:

- $e^+e^- \rightarrow t\bar{t}$:

- $yyxye\nu$, $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \mp 1$: <https://ild.ngt.ndu.ac.jp/elog/opt-data/?GenProcessID=108670>

This sample contains the final state resulting from the $W \rightarrow e\nu$ decay.

- $yyxye\nu$, $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \pm 1$: <https://ild.ngt.ndu.ac.jp/elog/opt-data/?GenProcessID=108670>

This sample contains the final state resulting from the $W \rightarrow e\nu$ decay.

- $yyxy\nu\ell$, $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \mp 1$: <https://ild.ngt.ndu.ac.jp/elog/opt-data/?GenProcessID=108675>.

This sample contains the final state resulting from the $W \rightarrow \ell\nu$ decay with $\ell = \mu, \tau$. For the analysis presented here the final state with $\ell = \tau$ has been discarded.

- $yyxy\nu\ell$, $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \pm 1$: <https://ild.ngt.ndu.ac.jp/elog/opt-data/?GenProcessID=108676>.

This sample contains the final state resulting from the $W \rightarrow \ell\nu$ decay with $\ell = \mu, \tau$. For the analysis presented here the final state with $\ell = \tau$ has been discarded.

- $e^+e^- \rightarrow b\bar{b}$: The $b\bar{b}$ events are extracted from a sample of simulated 2-quark events <https://ild.ngt.ndu.ac.jp/elog/opt-data/?GenProcessID=250114>. The generated cross section for these 2-quark events is 32470 fb and the total integrated luminosity is about 46 fb^{-1} . The genuine $e^+e^- \rightarrow b\bar{b}$ at 250 GeV is about 4% of the total cross section yielding a total number of events of about 60000.

Note that the analysis of the $e_R^+e_L^- \rightarrow t\bar{t}$ came quite late in the study since it is generally considered that the (semi-leptonic) $e_L^+e_R^- \rightarrow t\bar{t}$ is the more challenging channel. Therefore the corresponding control plots in Secs. 3 and 5 are only given for this channel in this note.

3. Efficiencies and Control plots

Figure 1 demonstrates that the studies presented in this note test the detector performance for very different momenta of the final state b quark.

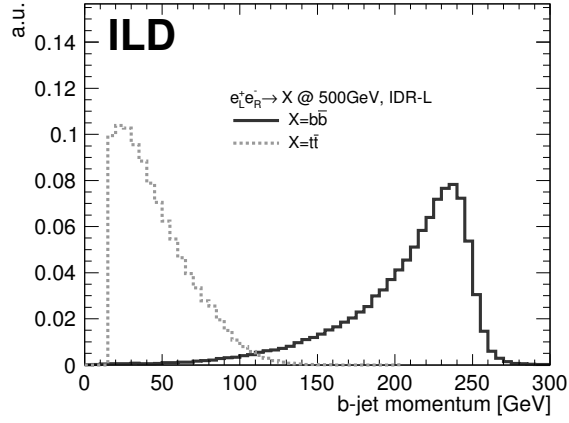


Figure 1: Momentum of the b -jet with cheated identification for $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow t\bar{t}$ processes.

105 The Figs. 2 and 3 show the missed tracks before and after vertex recovery for the $e^+e^- \rightarrow b\bar{b}$ and
 106 $e^+e^- \rightarrow t\bar{t}$ analyses, respectively. Both figures suggest a systematic improvement in the assignment of
 107 secondary vertices.

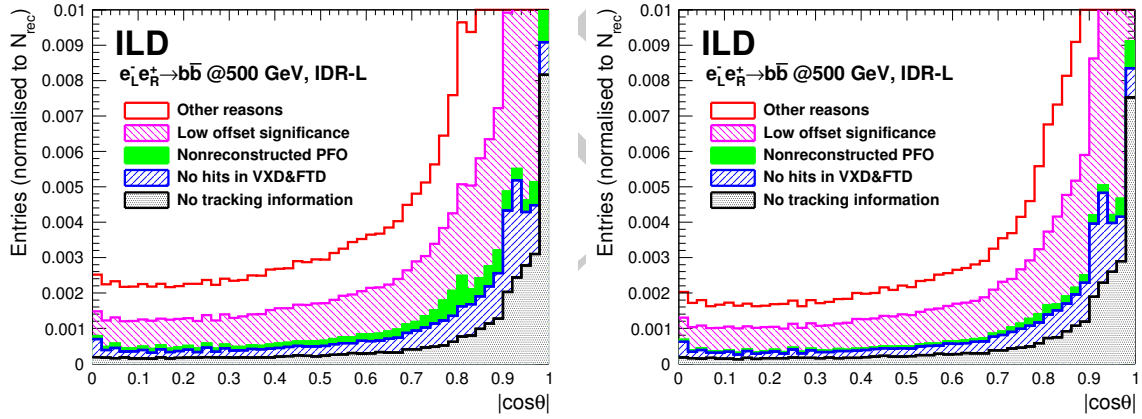


Figure 2: Vertex recovery in case of the $e^+e^- \rightarrow b\bar{b}$ process.

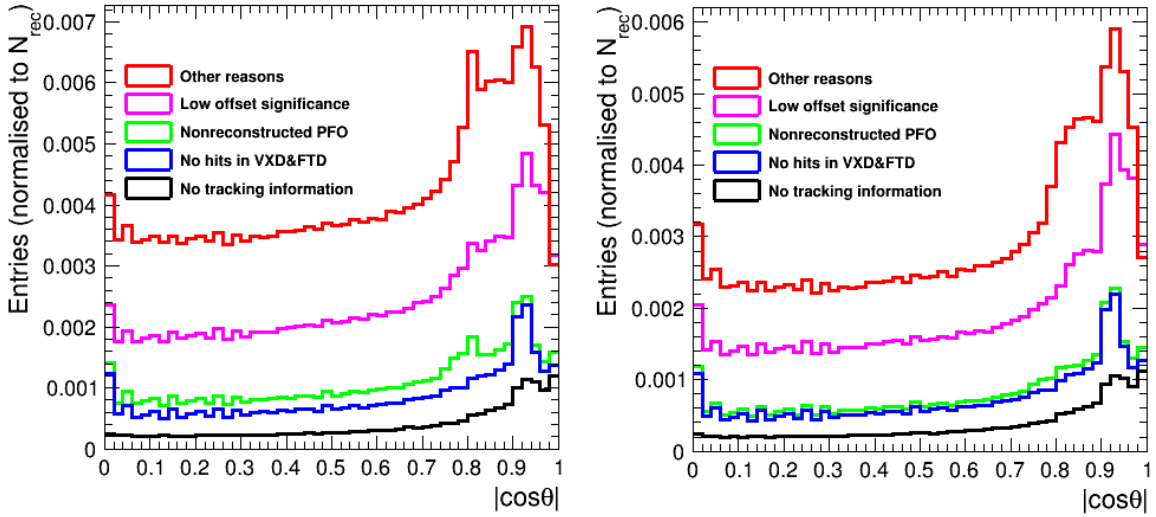


Figure 3: Vertex recovery in case of the $e^+e^- \rightarrow t\bar{t}$ process.

108 This improvement is quantified in Figs. 4 and 5 where the purity of the b -charge reconstruction
 109 is shown as a function of the b -tag value, the reconstructed b -momentum $|p_{had}|$ the number of
 110 reconstructed tracks assigned to a secondary vertex N_{rec} and finally the polar angle of the b -hadron.
 111 here denoted as $|\cos\theta|$. The b -charge purity is defined as

$$p_b = N_{correct}/N_{jet,tot}. \quad (4)$$

112 with $N_{correct}$ being the number of b -jets with correctly reconstructed b -quark charge. This value nor-
 113 malised to the total number of b -jets $N_{b-jet,tot}$. for which a charge assignment according to e.g. Table 2
 114 can be made.

115 The improvement is larger for the process $e^+e^- \rightarrow t\bar{t}$ than for $e^+e^- \rightarrow b\bar{b}$. Qualitatively this is
 116 expected since as a consequence of the different b -jet momenta, see Fig. 1, also the tracks produced
 117 in the decay of the b -hadron are softer in case of top-pair production. In case of $e^+e^- \rightarrow t\bar{t}$ the
 118 improvement is 10% over a large range in $|\cos\theta|$ and mainly driven by three to five prong decays.
 119 Both results will further improve once the vertex recovery takes also the the impact parameter z_0 into
 120 account. All results shown so far in this section have been obtained for the large detector model. The
 121 conclusions for the small detector model are similar.

122 The lower right panels of Figs. 4 and 5 show a drop in purity for large values of $|\cos\theta|$. This is
 123 compatible with the drop in acceptance that is shown in Fig. 6 for the case $e^+e^- \rightarrow b\bar{b}$ as a function
 124 of the polar angle of the reconstructed b -jet $|\cos\theta_b|$. Within statistical errors the results are the same
 125 for the large and the small detector model. However, towards large values of $|\cos\theta_b|$ the large detector
 126 performs systematically better than the small detector.

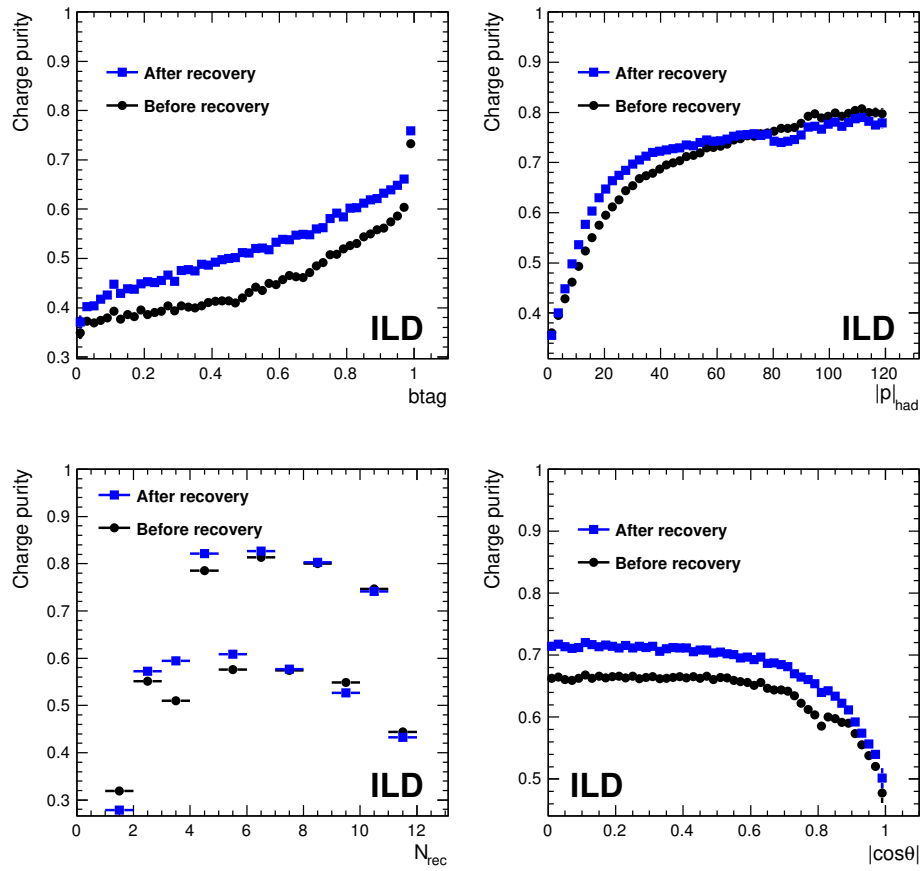


Figure 4: Purity before and after vertex recovery in case of the $e^+e^- \rightarrow t\bar{t}$ process for different observables.

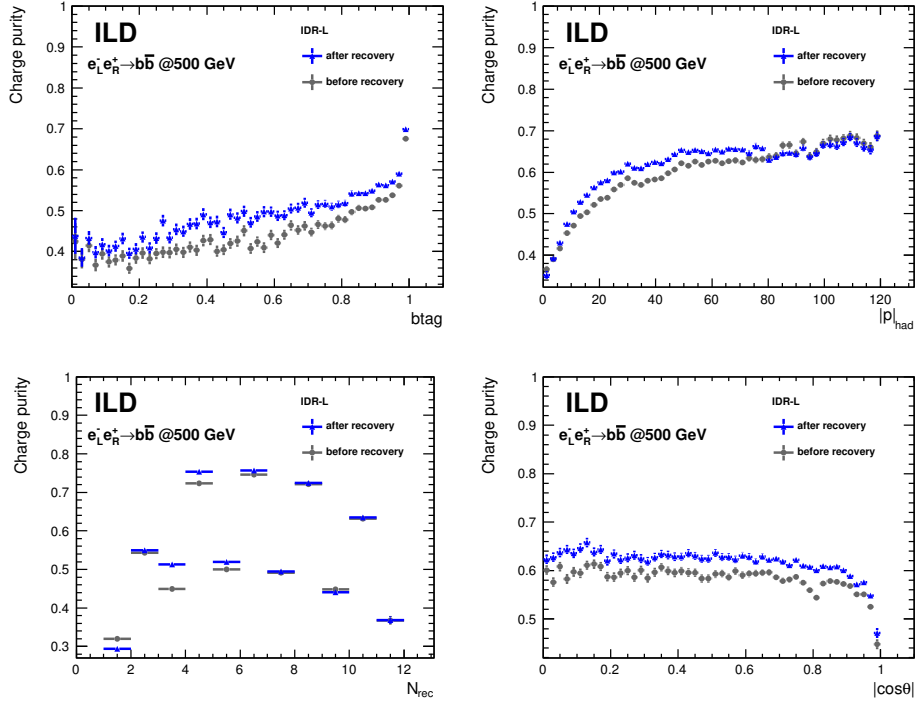


Figure 5: Purity before and after vertex recovery in case of the $e^+e^- \rightarrow t\bar{t}$ process for different observables.

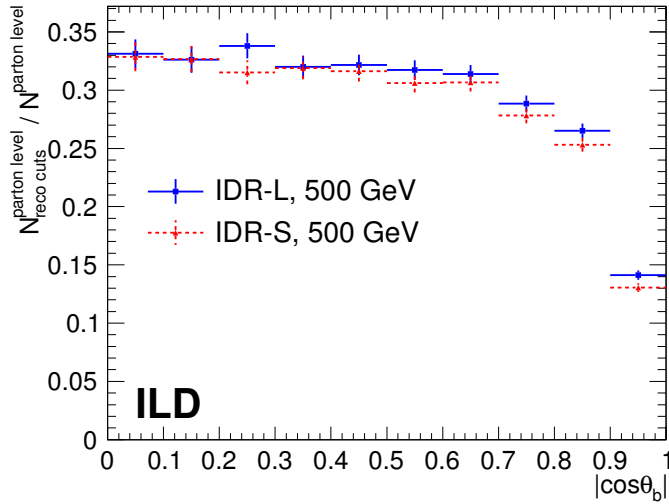


Figure 6: Detector acceptance distribution for b-tagged jets. Note the figure shows the acceptance after the full selection given in Tab. 1.

127 A component that distinguishes the ILD Detector from other proposals for e^+e^- colliders is the TPC
 128 as the central tracking system. Beside the precise momentum measurement the dE/dx measurement
 129 in the gaseous medium allows for a particle identification. Since around 80% of B-Mesons (neutral or
 130 charged) contain a charged Kaon among their decay products the particle ID can support greatly the

131 charge determination of the b -quark. The Fig. 7 displays the normalised dE/dx spectrum for different
 132 particles in different momentum ranges for the large and the small detector model. In both cases there
 133 is a clear separation of Kaons from pions. The latter are however much more abundant. There is only
 134 a small population of protons. Figure 8 shows the dE/dx spectra for the two processes under study.
 135 Finally the Fig. 9 shows the variation of the purity as as a function of the Kaon selection efficiency.

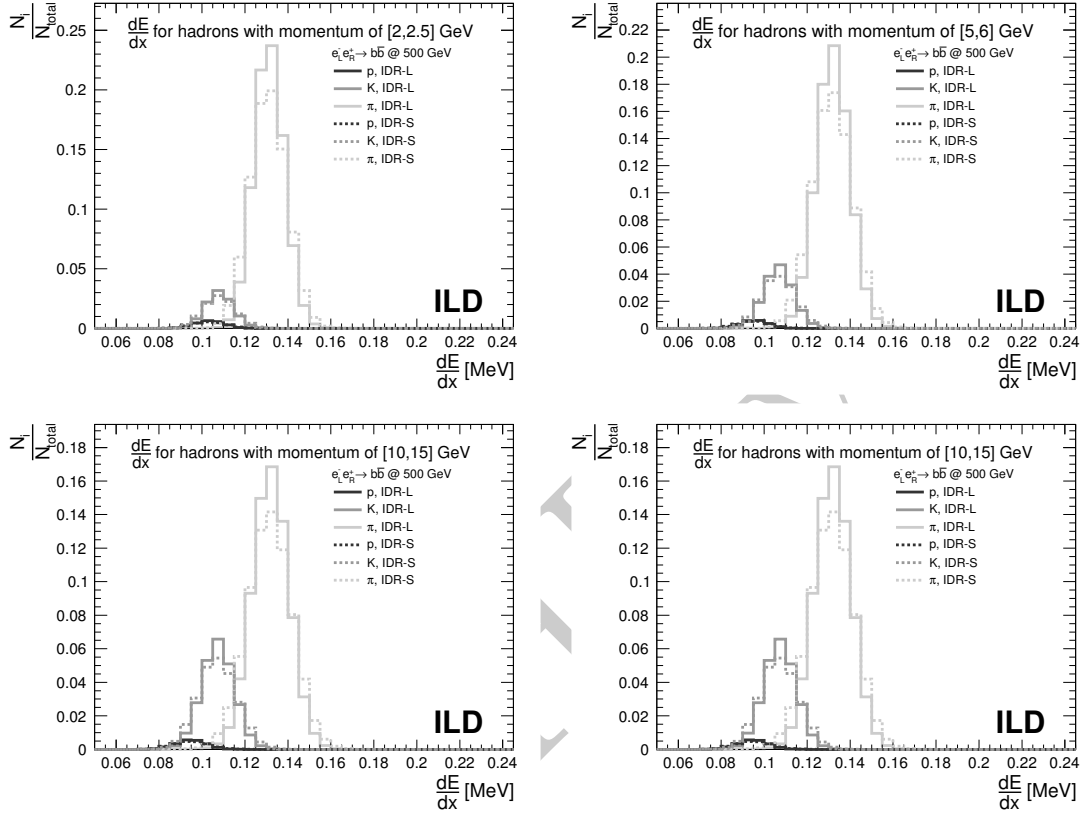


Figure 7: Projection of dE/dx for several momentum ranges. Comparison of hadron separation performance by different detector models in $b\bar{b}$ final states.

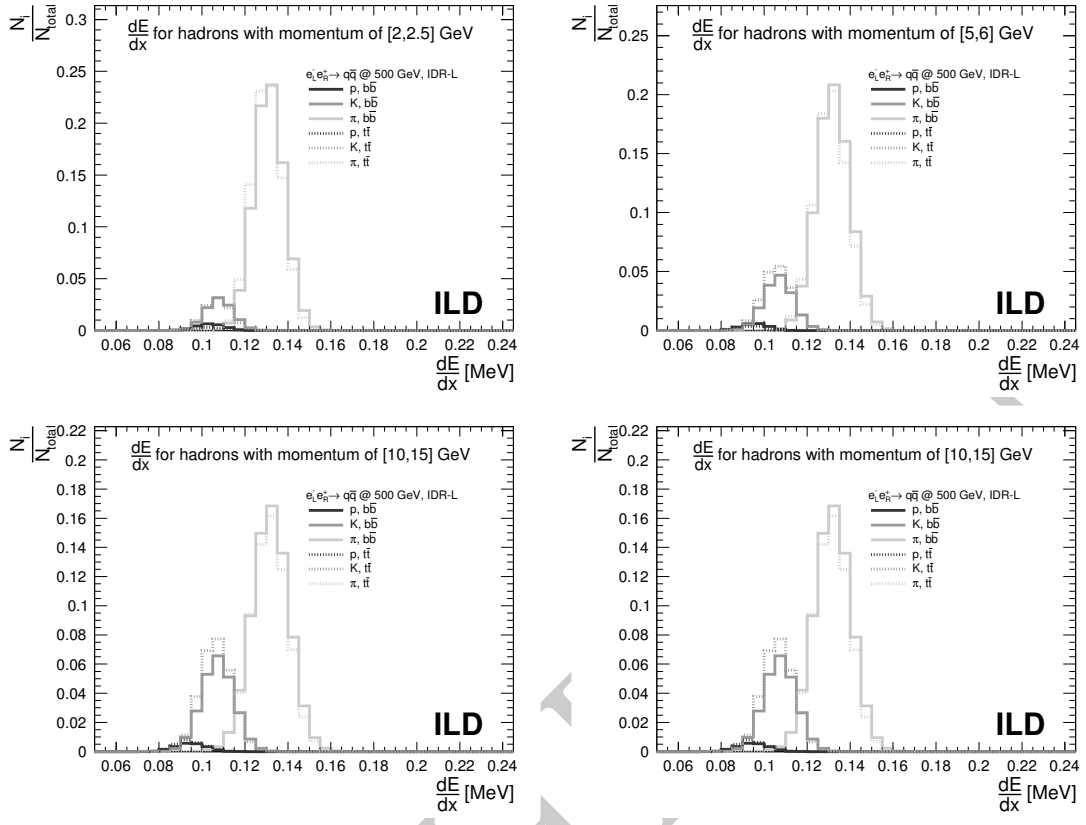


Figure 8: Projection of dE/dx for several momentum ranges. Comparison of hadron separation performance by the large model for different topologies.

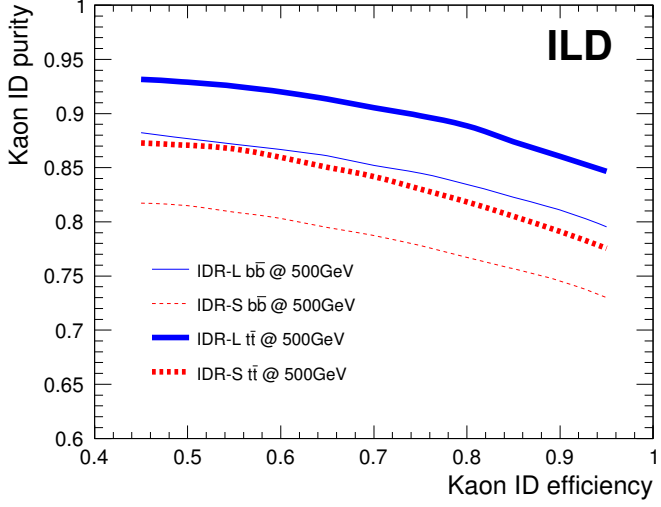


Figure 9: Purity of the Kaon selection as a function of the Kaon selection efficiency.

$$e_L^- e_R^+ \rightarrow b\bar{b} \text{ at } 500 \text{ GeV}$$

	IDR-L			IDR-S		
	Signal	B _{q\bar{q}} /S	B _{rad.Z} /S	Signal	B _{q\bar{q}} /S	B _{rad.Z} /S
Full sample	100.0%	1800.5%	359.1%	100.0%	1800.6%	359.0%
$b_{tag}(jet_1) > 0.9$ and $b_{tag}(jet_2) > 0.2$	70.2%	2.3%	147.7%	69.9%	2.3%	149.0%
$m_{jet_1+jet_2} > 200 \text{ GeV}$	68.2%	1.4%	6.7%	67.8%	1.2%	6.7%
$E_{photon} < 100 \text{ GeV}$	64.8%	1.3%	1.7%	64.3%	1.2%	1.6%
double jet-charge measurement	28.9%	1.0%	1.0%	27.9%	0.9%	1.0%

Table 1: Selection efficiency and B/S rejection for some bkg sources

$$e_L^- e_R^+ \rightarrow b\bar{b} \text{ at } 500 \text{ GeV}$$

	IDR-L	IDR-S
Vtx+Vtx	12.9%	12.8%
K+K	4.4%	4.0%
Vtx+K (diff. jets)	3.9%	3.7%
Vtx+K (same jet)	7.7%	7.4%

Table 2: Final selection efficiency, after double jet-charge measurement

136 4. Analysis details specific to the $e_L^- e_R^+ \rightarrow b\bar{b}$ analysis

137 Table 1 shows the selection efficiencies for the $e_L^- e_R^+ \rightarrow b\bar{b}$ analysis. The overall efficiency is with
138 around 28% to 29% similar for both detector models. For the b -charge measurement opposite charges
139 in opposite jets are required. The charges are either derived from the tracks pointing to the secondary
140 vertex or from the Kaon charge or from a combination of both. The efficiencies for the different methods
141 are given in Tab. 2. The purity of the different methods is shown in Fig. 10. In both cases there is no
142 large difference between the two detector models although the large detector seem to perform slightly
143 better for the double Kaon method.

144 5. Analysis details specific to the $e^+ e^- \rightarrow t\bar{t}$ analysis

145 Figure 11 shows in the left panel the energy distribution of the isolated lepton in the laboratory
146 frame. The distribution features a maximum at around 30 GeV and a tail towards higher energies. The
147 right panel shows the polar angle spectrum of the isolated lepton. The distribution decreases slightly
148 with a sharp drop at the acceptance limit of the detector. The distribution reveals also acceptance
149 drops at $\cos \theta_\ell = 0$ and $\cos \theta_\ell = 0.8$ that correspond to the position of the TPC anode plate and the
150 barrel-endcap transition region, respectively.

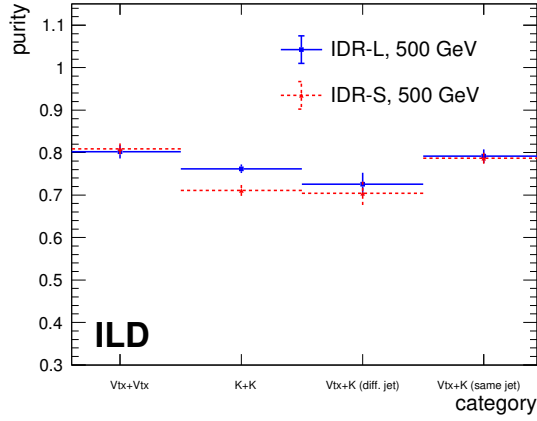


Figure 10: Purity of the different methods

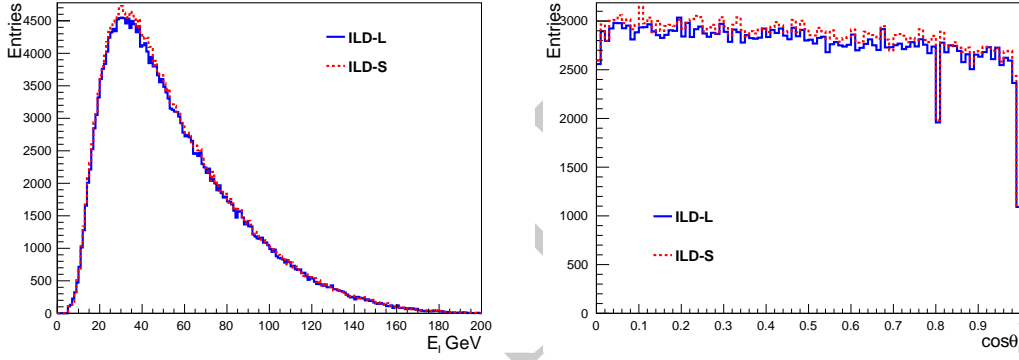


Figure 11: Left: Energy of the isolated lepton in $e_L^+ + e_R^- \rightarrow t\bar{t}$. Right: Polar angle distribution of the isolated lepton in $e_L^+ e_R^- \rightarrow t\bar{t}$.

151 For completeness Fig. 12 shows the mass distribution of the hadronic W and the hadronic top
 152 quark.

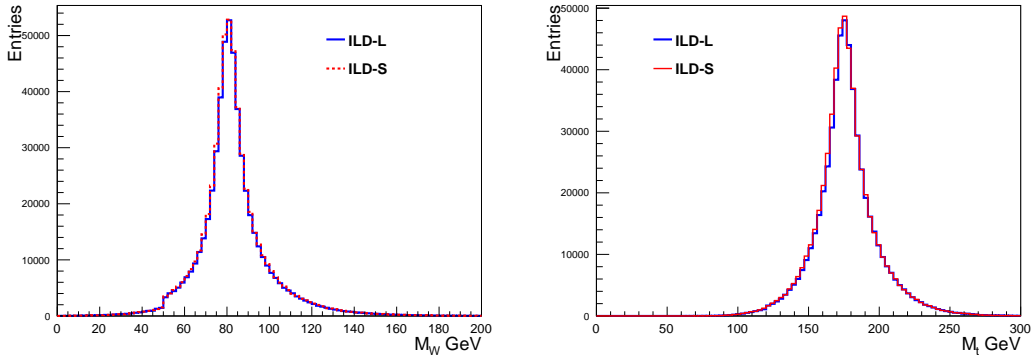


Figure 12: Left: Mass distribution of the hadronic W in $e_L^+ + e_R^- \rightarrow t\bar{t}$. Right: Mass distribution of the hadronic top in $e_L^+ e_R^- \rightarrow t\bar{t}$.

153

Tables 3 and 4 give the efficiencies after each cut applied for the selection of $t\bar{t}$ events.

$e_L^- e_R^+ \rightarrow t\bar{t}$ at 500 GeV		
	IDR-L	IDR-S
Isolated Lepton	92.1%	92.1%
$btag_1 > 0.8$ or $btag_2 > 0.3$	81.2%	81.1%
Thrust < 0.9	81.2%	81.1%
Hadronic mass	78.2%	78.2%
Reconstructed m_W and m_t	73.4%	73.4%

Table 3: Event selection efficiencies after each pre-selection criteria presented. Note that this does not concerns any background effects.

$e_R^- e_L^+ \rightarrow t\bar{t}$ at 500 GeV		
	IDR-L	IDR-S
Isolated Lepton	94.1%	94.0%
$btag_1 > 0.8$ or $btag_2 > 0.3$	84.9%	84.8%
Thrust < 0.9	84.9%	84.8%
Hadronic mass	82.2%	82.3%
Reconstructed m_W and m_t	77.6%	77.5%

Table 4: See Tab. 3 for details.

154

Figure 13 shows the fraction of accepted events, see Eq. 2 for different methods used to distinguish the top from the anti-top. The methods are the same as those given in Tab. 2. The semi-leptonic decay of the top quark gives an additional powerful information for the event reconstruction giving rise to the additional variable L_{cut} , which means the charge of the isolated lepton plus a cut on the event quality $\chi^2 < 15$ that is motivated in Ref. [1].

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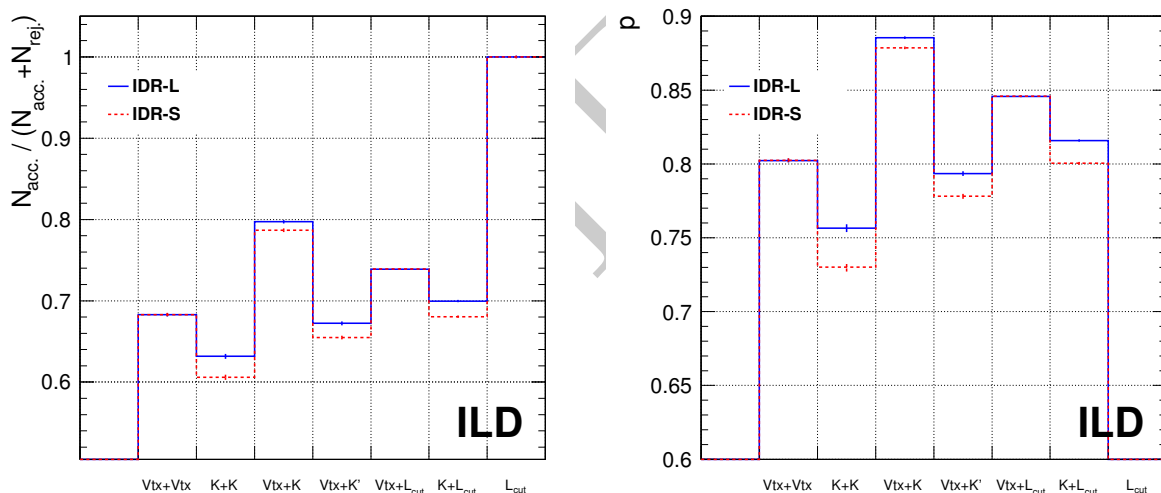


Figure 13: Calculated probability plots for $t\bar{t}$ events. Left: The result of $N_{acc.}/(N_{acc.} + N_{rej.})$. Right: Purities for different charge configurations. **THE LEFT PART OF THIS FIGURE STILL NEEDS REVISION.**

158

159 6. Results

160

Figure 14 shows the polar angle spectrum after the application of Eq. 2 for the $e_L^- e_R^+ \rightarrow b\bar{b}$. Large and small detector agree within statistical uncertainties. It seems however that there is larger migration for the small detector.

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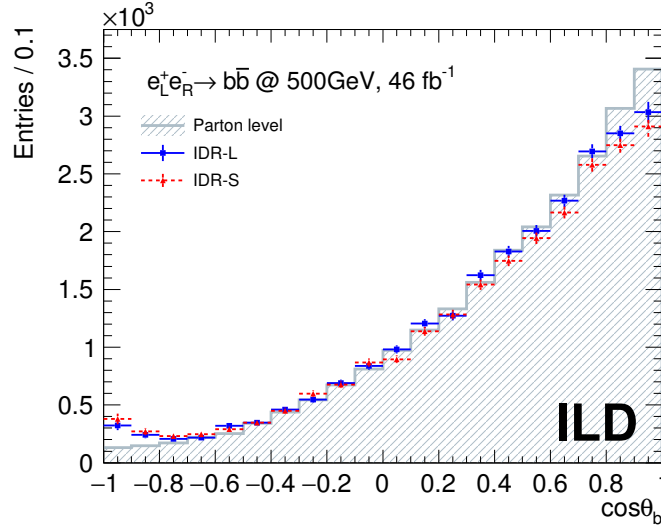


Figure 14

163 The left part of Fig. 15 shows the polar angle distribution of $t\bar{t}$ of the generated and reconstructed
 164 data for the large and the small detector models. For this all methods introduced in Fig. 13 have been
 165 applied. In addition events have been selected for which the momentum of the reconstructed B -hadron
 166 is greater than 15 GeV whenever the result of the vertex charge is used for a jet and the sum of the
 167 Lorentz γ of the top-quark pair is required to be greater than 2.4. The red dotted line shows the fitted
 168 result of the reconstructed events. The right part shows the polar angle distribution of the underlying
 169 b -quark for the same set of cuts.

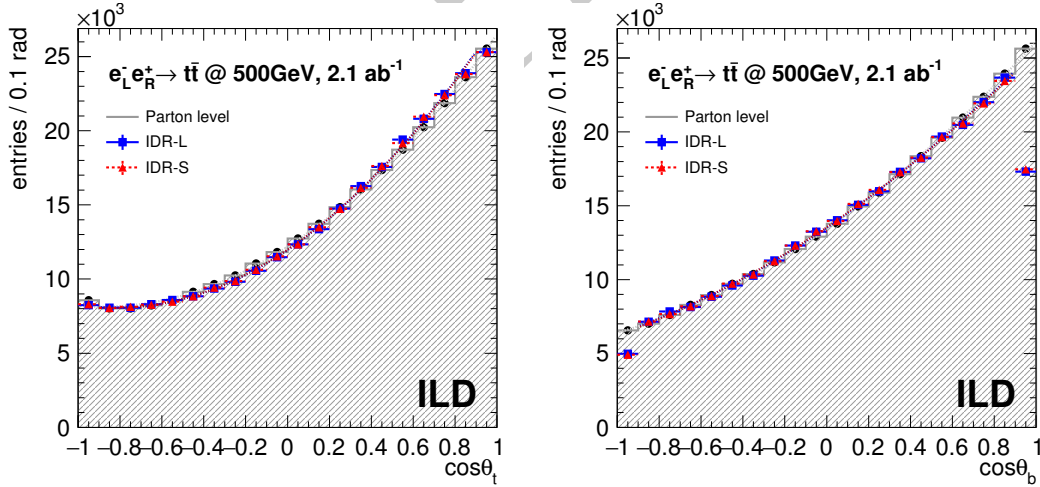


Figure 15: Results for $e_L^- e_R^+ \rightarrow t\bar{t}$ Left: Polar angle distribution for top quark. Right: Polar angle distribution for the b -quark that is issue of the top-quark decay. The distributions for IDR-S is normalised to the one for IDR-L so that both histograms will be on the same level.

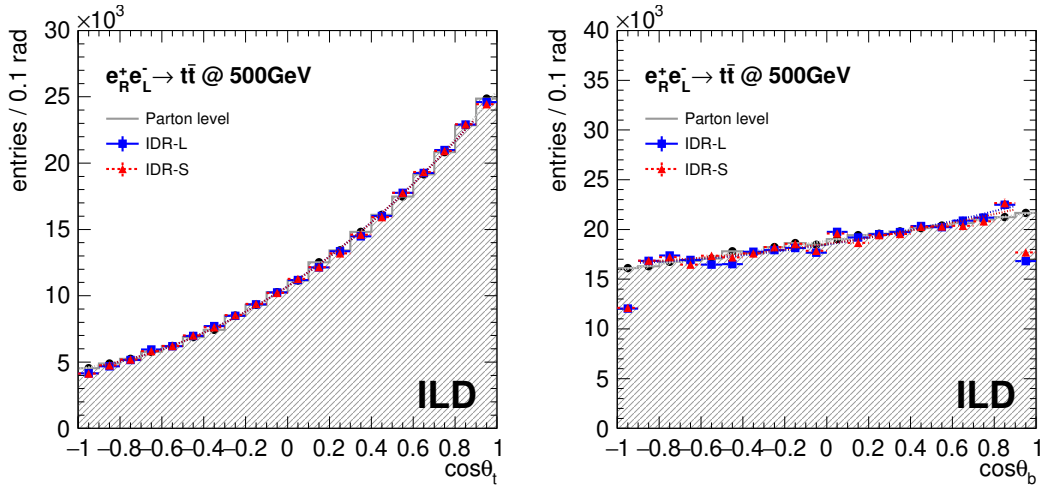


Figure 16: Same as Fig. 15 $e_R^- e_L^+ \rightarrow t\bar{t}$.

170 The left part of figure 17 shows the polar angle spectrum for the case $e_R^- e_L^+ \rightarrow t\bar{t}$. For this the
 171 events remaining after the cuts in Table 4 are only subject to a further cut on the sum of the Lorentz
 172 γ of the top-quark pair of 2.4 as in the case of the $e_L^- e_R^+ \rightarrow t\bar{t}$ channel. The generated spectrum can
 173 be reconstructed very precisely. The right hand part of Fig. 17 shows the polar angle spectrum of the
 174 underlying b -quark. Here only events with consistent vertex-charge measurements have been included.
 175 The polar angle of the b -quark can also in this case be very well reconstructed. However, the efficiency
 176 drops to 10%. The inclusion of the other methods is subject to further studies. Preliminary results
 177 show that taking into account the isolated lepton “swamps” the polar angle spectrum with events in
 178 which the b -quark direction is not constrained. One reason is certainly that in the case of $e_R^- e_L^+ \rightarrow t\bar{t}$
 179 the b -quark is on average softer than in the case of $e_L^- e_R^+ \rightarrow t\bar{t}$.

180 Comparing the spectra of the underlying b -quarks demonstrate more clearly than the actual top-
 181 polar angle spectra the different polarisation of the top quarks projected out by the flight direction
 182 of the b -quark. In case of $e_L^- e_R^+ \rightarrow t\bar{t}$ the final state is enriched with left-handed top-quarks. In this
 183 case the b -quark is preferably emitted in the direction of the top-quark. Therefore the two polar angle
 184 spectrum in Fig. 15 looks similar to each other. In case of $e_L^- e_R^+ \rightarrow t\bar{t}$ In case of $e_R^- e_L^+ \rightarrow t\bar{t}$ the final
 185 state is enriched with right-handed top-quarks. In this case the b -quark is preferable emitted opposite
 186 to the direction of the top-quark. The polar angle of the b -quark is a consequence of the Jacobian
 187 peak in the vicinity of $\cos \theta_{Wb} = 0$, with $\cos \theta_{Wb}$ being the opening angle between the b -quark and the
 188 W -boson, generated by the boost of the back-to-back configuration in the centre-of-mass frame of the
 189 decaying top into the laboratory frame.

190 Table 5 lists the overall selection efficiencies and the generated and reconstructed value of the
 191 forward-backward asymmetry $A_{FB, reco}^t$ as an estimator for the quality of the reconstruction.

	$e_L^- e_R^+ \rightarrow t\bar{t}$		$e_R^- e_L^+ \rightarrow t\bar{t}$	
	IDR-L	IDR-S	IDR-L	IDR-S
$A_{FB, gen}^t$	0.329		0.430	
$A_{FB, reco}^t$	0.342	0.340	0.430	0.430
$\varepsilon_{A_{FB, reco}^t}$	0.306	0.305	0.641	0.641

Table 5: Selection efficiencies and resulting $A_{FB, reco}^t$ for both beam polarisations and the two detector models under study.

192 So far the results have been presented for full beam polarisation. Using the known formula [4]

$$\sigma_{\mathcal{P}_{e^-}, \mathcal{P}_{e^+}} = \frac{1}{4} [(1 - \mathcal{P}_{e^-} \mathcal{P}_{e^+})(\sigma_{-,+} + \sigma_{+,-}) + (\mathcal{P}_{e^-} - \mathcal{P}_{e^+})(\sigma_{+,-} - \sigma_{-,+})], \quad (5)$$

193 with $\sigma_{-,+}$ and $\sigma_{+,-}$ being the fully polarised cross-sections, the results can be extrapolated to the
 194 realistic beam polarisations of $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = \mp 0.8, \pm 0.3$. The resulting Born level cross sections are
 195 1070 fb^{-1} in case of $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = -0.8, +0.3$ and 519 fb^{-1} in case of $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = +0.8, -0.3$. The
 196 resulting statistical errors for an integrated luminosity of $\mathcal{L} = 1600 \text{ fb}^{-1}$ at each of the two polarisation
 197 settings are given in Table 6.

	$\mathcal{P}_{e^-}, \mathcal{P}_{e^+}$	$(\delta\sigma/\sigma)_{stat.} [\%]$	$(\delta A_{FB}^t/A_{FB}^t)_{stat.} [\%]$
IDR-L	-0.8, +0.3	0.17	0.7
	+0.8, -0.3	0.25	0.53
IDR-S	-0.8, +0.3	0.17	0.7
	+0.8, -0.3	0.25	0.53

Table 6: Statistical precisions expected for the cross sections and A_{FB}^t for the case $\mathcal{P}_{e^-}, \mathcal{P}_{e^+} = -0.8, +0.3$ and the two detector models under study.

198 For both, the cross section and the forward backward asymmetry it can be expected that even at
 199 full luminosity the statistical error has to be taken into account. However, the systematic errors need
 200 to be carefully estimated. For the present analysis it would have to be checked how much the sample
 201 is contaminated by events for which the semi-leptonic decay yields τ -leptons or the τ -leptons are taken
 202 into account as in [1]. The contamination by fully hadronic $t\bar{t}$ events can be expected to be small.

203 To put the results into context, the precisions on the cross-sections and the forward-backward
 204 asymmetries are translated into precisions on electromagnetic form factors of the top quark. Figure 17
 205 shows the precisions at the 1σ level expected at ILC500. The precisions are compared with those
 206 expected after the full HL-LHC running and estimations produced for FCC-ee [5] at the same confidence
 207 level. For ILC500, the two sets F_1 and F_2 have been extracted separately but within each set the
 208 uncertainties have been extracted simultaneously. The projections for HL-LHC are derived from the
 209 *individual* constraints of EFT Wilson coefficients presented in Tab. C2.3 of Ref. [6] (the most favorable
 210 scenario for HL-LHC). These figures demonstrate clearly the superiority of a linear e^+e^- collider with
 211 polarised beams operated at an adequate centre-of-mass energy.

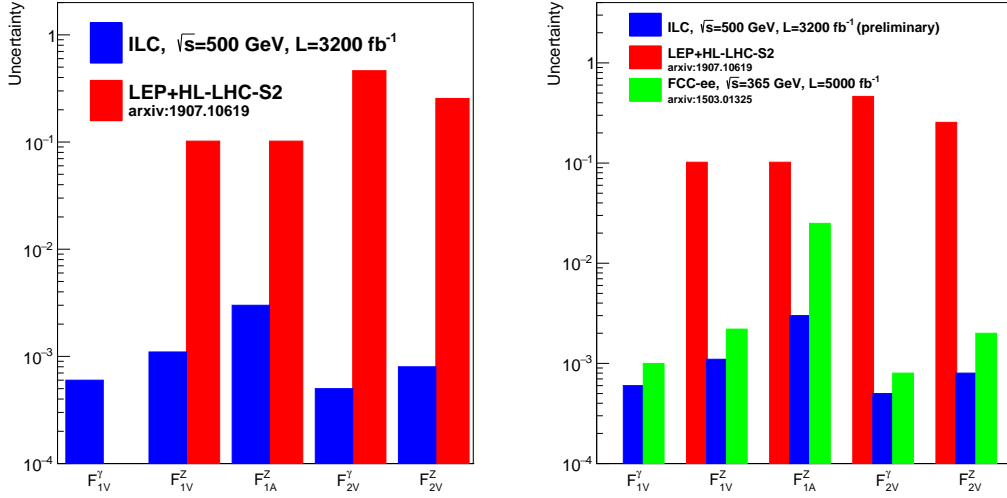


Figure 17: Precisions the on electromagnetic form factors expected after ILC500 compared with those expected after the full HL-LHC running and an estimation for FCC-ee after 5000 fb⁻¹. See text for further details.

212 7. Summary

213 The analysis shows that both, the large and the small detector model are capable to provide a high
 214 precision measurement of the cross-section and the polar angle spectrum of semi-leptonic $t\bar{t}$ events.
 215 The statistical precision on the cross section and the forward backward asymmetries are compatible
 216 with the results found in [1]. For the first time the underlying polar angle spectrum of the b -quark
 217 issue of the top quark decay is presented. This spectrum reveals more clearly the acceptance drop
 218 towards large polar angles. Still the polar angle of the b -quark can be reconstructed accurately within
 219 $\cos\theta < 0.8$ in case of $e_L^- e_R^+ \rightarrow t\bar{t}$. The generated and reconstructed spectra are tilted for the case
 220 $e_R^- e_L^+ \rightarrow t\bar{t}$. Here further work is clearly needed. Future studies should include the application of Eq. 2
 221 for the $t\bar{t}$ case and a revision of the vertex recovery (list not exhaustive). However, both cases allow
 222 still for the perspective that in the future the fully hadronic final state can be taken into account for
 223 the analysis. It is likely that this will increase yet more the precision on the form factors shown in
 224 Fig. 17.

225 The study of the $t\bar{t}$ production has been accompanied by the second benchmark study $e^+e^- \rightarrow b\bar{b}$.
 226 Since only 46 fb⁻¹ are available for this channel it should be rather considered as an auxiliary study.
 227 However, it is shown that the polar angle spectrum can be very well reconstructed even for the hard
 228 b -jets and that migrations can be controlled at a satisfactory level. It is therefore justified to conclude
 229 that ILD should be able to make precision measurements of this channel even at a centre-of-mass
 230 energy of 500 GeV. Although the results are similar for both the small and the large detector model,
 231 this analysis, more than the analysis of the $t\bar{t}$ process reveals a slight preference for the large model.

232 Also in the future all heavy quark studies should be carried out in close cooperation with each
 233 other. As can be seen from the present study there are many common issues between the studies.
 234 In the future emphasis will be put on systematic uncertainties given e.g. by hemisphere correlations.
 235 These studies may be more involved in case of $t\bar{t}$ since the heavy quarks are in general not back-to-back.

236 **Acknowledgements**

237 **References**

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251 **Appendix**

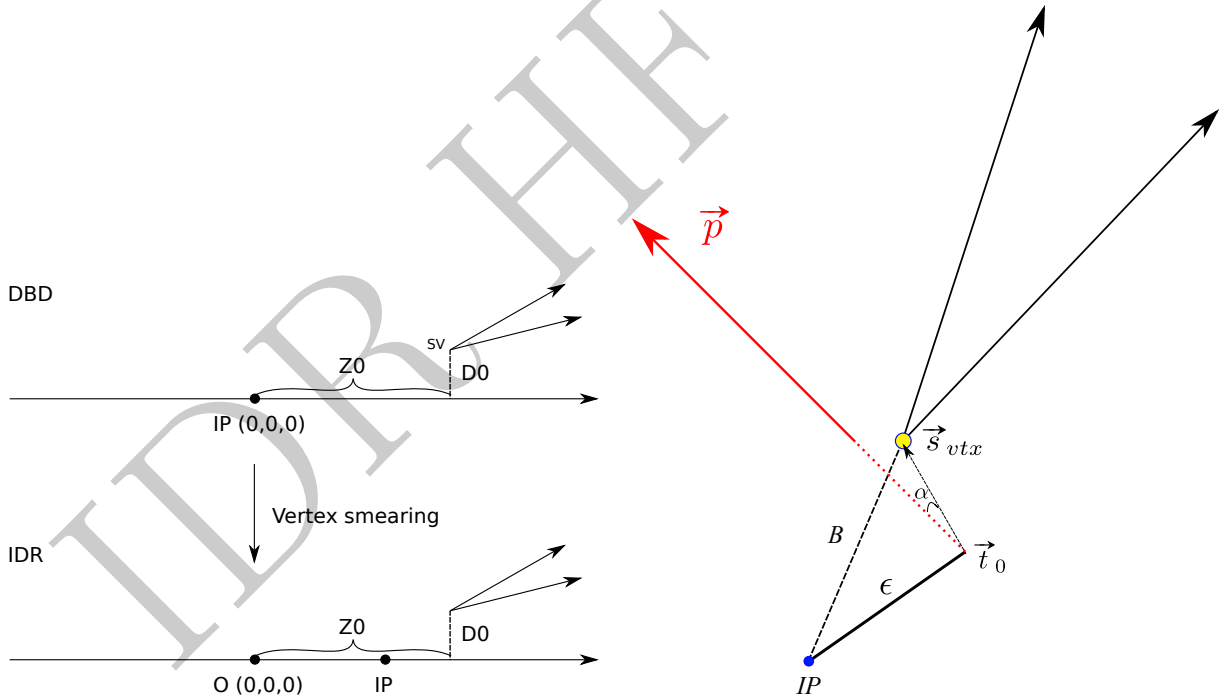


Figure 18: Left is