Heavy Quarkonium

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GENERAL INTRODUCTION

As a theory subject to asymptotic freedom QCD is characterized by an intrinsic mass scale, $\Lambda_{QCD} \sim 200-400$ MeV. It is around this scale where all perturbative calculations for the Q^2 evolution of the strong coupling constant $\alpha_s = q_s^2/(4\pi)$ diverge. Accordingly, Λ_{QCD} is the dividing scale between the perturbative and the non-perturbative regime. Because of this the physics of bound systems of light quarks — quarks with masses below Λ_{QCD}, namely up, down, strange — is expected to be a lot more complex than the physics of bound systems of the heavy quarks charm and bottom [**?**]. In fact, it follows from asymptotic freedom that at large energy scales/small distances the quark–gluon interaction gets Coulombic. Accordingly the spectrum of low lying heavy quarkonia is very similar to that of positronium, as illustrated in Fig. 1. Since QCD keeps colored objects confined within bound systems that are overall color neutral, the potential needs to deviate from the Coulomb potential as the distance between the bound objects increases, for the potential needs to keep rising. Already in the 1970ies the Cornell group found that a central potential of the kind

$$V(r) = -\alpha/r + \sigma r \ , \tag{1}$$

where σ denotes the so-called string tension, supplemented with property adjusted spin and angular momentum terms, allows for a perfect description of all states known at the time and even until quite recently (see, e.g. the review presented in Ref. [1]). Using (p)NRQCD — the effective field theory for heavy quarkonia, to be explained in the next section — and lattice QCD Eq. (1) later received a sound foundation within QCD.

It therefore came as a big surprise when with the discovery of $X(3872)$ in 2003 a state was found experimentally that did not at all fit into the predictions by the up to that time very successful quark model. Quickly the state was identified as a candidate for a QCD exotic — a meson with a substructure different from $\bar{Q}Q$. The $X(3872)$ did not stay alone. The Review of Particle Physics by the Particle Data Group [2] in its 2014 issue lists 8 established states that apparently can not be put into the quark model scheme. Most prominent amongst those are the charged states found in both the charmonium as well as the bottomonium mass range that decay exclusively into final states that contain a heavy quark and a heavy antiquark. Thus these states have to contain at least 4 quarks — they are explicitly exotic. As described below in a lot more detail the various models for exotic states can be classified according to their clustering of the quarks:

- if the heavy quark–antiquark pair is forming a compact, quarkonium like, core surrounded by the light quarks, the state is called **hadrocharmonium;**
- if a the light and the heavy quark as well as the light and the heavy anti–quark combine to form compact diquark and anti-diquark substructures, respectively, one speaks of **tetraquarks** (note, this applies to the most prominent tetraquark model, but there are also tetraquark models that do not assume any diquark clustering) and
- if the heavy quark and the light antiquark as well as the heavy antiquark as well as the light quark combine to form a meson pair the object is called **hadronic molecule**. When located close to the threshold of the molecular constituents the molecules can become quite extended — a feature that is claimed to lead to observable consequences.

In reality all the physical wave functions might contain some fraction of all of these configurations and at least the neutral ones even an admixture of regular quarkonium. At present it is the main goal of research in the field to identify the most prominent component — this enterprise calls for refined theories that allow one to relate observables to the underlying substructures in a controlled way as well as experiments of sufficient quality and quantity to be decisive.

HADRONIC TRANSITIONS

REGULAR QUARKONIA - OPEN ISSUES AND CHALLENGES

*** To be done ***

QCD EXOTICS

The spectrum of the currently established charmonium like states is depicted in Fig. 2. As one can see, above

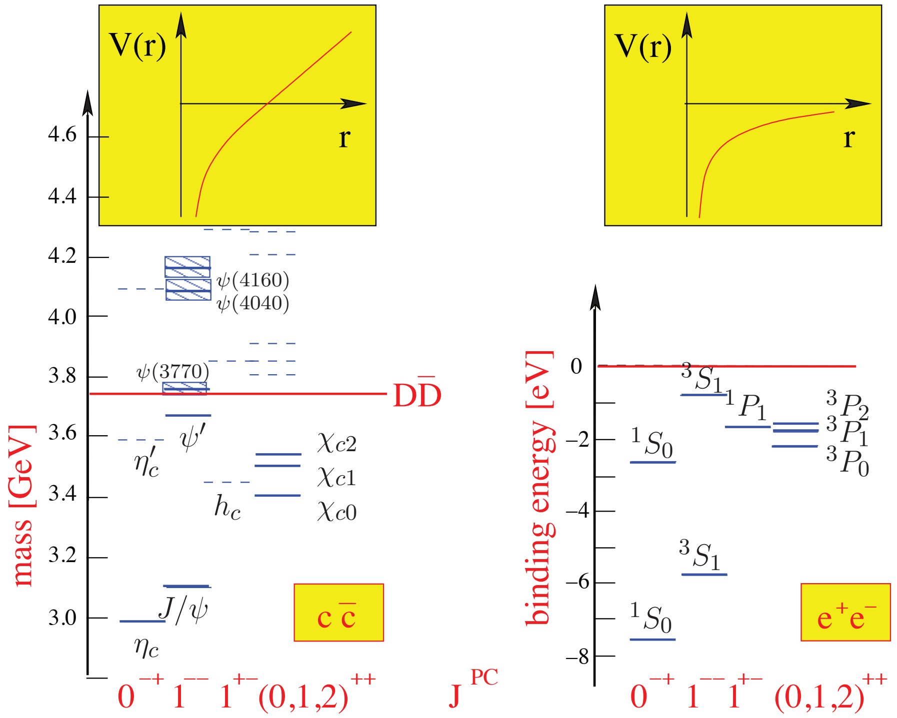

FIG. 1: Comparison of the charmonium potential as well as spectrum as it was known 2002, before the discovery of $X(3872)$ to those of positronium.

the lowest open charm threshold the number of exotics candidates already outnumbers the number of states that seem to match to the expectations of the quark model.

When restricting ourselves to confirmed states we are faced with 8 states that do not seem to fit into the standard quark model: The 4 established charged states ($Z_c(3900)^{\pm}$ and $Z_c(4430)^{\pm}$ in the charmonium sector, and $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ in the bottomonium sector) as well as 4 additional neutral ones ($X(3872)$, $Y(4260)$, $Y(4360)$, $Y(4660)$).

Before discussing the current level of understanding for the various states in detail, in the following we will briefly describe the most popular models proposed for their structure.

Models

Tetraquarks

*** To be done ***

Hadrocharmonia

Triggert by the observation that a large number of exotic candidates decay into a quarkonium accompanied by one or more pions, they were proposed in Ref. [3] to consist of a core provided by a heavy quarkonium surrounded by an excited state of light–quark matter. In this picture the mentioned decays are understood as setting free the quarkonium core in the process of de-exciting the light–quark cloud into one or more pions.

It is expected that the dominant decay modes of hadroquarkonia are given by light quarks in combination with the core quarkonium. In particular, since in heavy quark systems the spins of the heavy quarks and the total angular momentum of the light–quarks are conserved individually, any given state should decay either into a spin 1 or a spin 0 quarkonium, but not into both. However, this spin symmetry selection rule can be evaded by mixing [4], as will be discussed below.

An additional implication of spin symmetry is that one should expect for each hadroquarkonium spin partner which can be constructed simply by replacing the heavy quarkonium core by its spin partner(s) while leaving the cloud intact. In this way a rather rich spectrum of additional hadroquarkonium states can be predicted [5]. Testing these predictions provides crucial tests for the hadroquarkonium model.

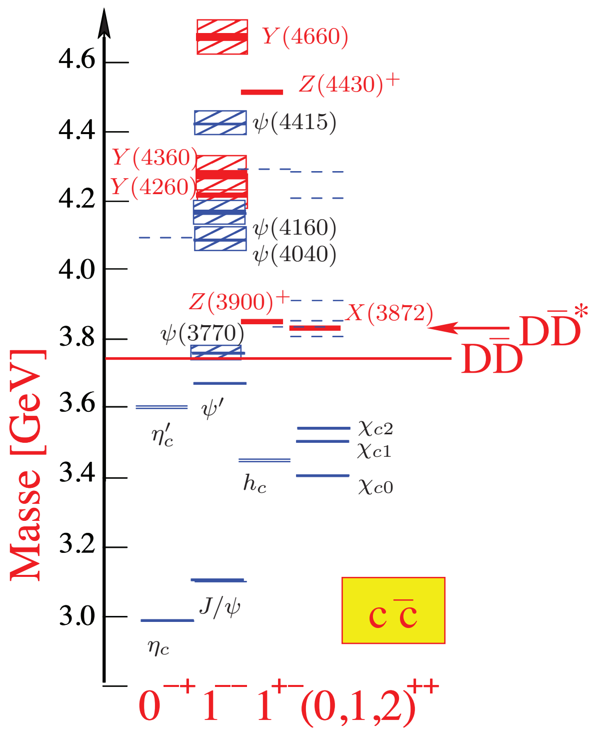

FIG. 2: Charmonium spectrum as of 2014. In blue are the states shown predicted by the quark model, while the red boxed show states that seem not to fit into the quark model predictions. The isovector states are marked by a ′+′ as superscript. Some thresholds, relevant for the discussion below, are also indicated.

Hadronic Molecules

Hadronic molecules are understood as bound states of two color neutral mesons. To get a quantum mechanical understanding of this statement one may think of the wave function of a physical state to be composed of two components: a two–hadron or molecular component and a compact component. Already in 1969 Weinberg showed that the probability to find the molecular component inside the physical wave function, λ^2, is related to the physical coupling of the state to the continuum channel via [6]

$$\frac{g_{\rm eff}^2}{4\pi} = 4(m_1+m_2)^2\lambda^2\sqrt{2\epsilon/\mu} \le 4(m_1+m_2)^2\sqrt{2\epsilon/\mu}\ , \quad (2)$$

where m_1, m_2 and μ denote the masses of the individual constituents and their reduced mass, respectively. The binding energy, ϵ, is defined with respect to the continuum threshold via

$$M = m_1 + m_2 - \epsilon\ , \quad (3)$$

where M denotes the mass of the state considered. Eq. (2) is correct up to corrections of the order of $\sqrt{2\mu\epsilon}R$, where R denotes the range of forces. Since $g_{\rm eff}^2$ is nothing but the residue at the pole for the state considered, via Eq. (2) the amount of molecular component in a wave function becomes an observable.

The derivation of Eq. (2) involves the normalization of a bound state wave function and it therefore holds rigorously only for stable bound states. However, it was shown that it can be generalized to states coupling to remote inelastic channels [7]. In addition, in order to keep the corrections small, the bound systems considered should be shallow. But one may adopt the physical picture that the coupling of a state gets large when it has a sizable molecular component also to somewhat more deeply bound systems — as was shown e.g. in Ref. [8] this assumption leads to quite significant observable consequences, like highly asymmetric line shapes.

While detailed predictions for new states within a molecular picture are difficult, since they require a detailed dynamical modeling analogous to that necessary to describe few nucleon systems, some general statements are still possible. For instance, molecules should form (predominantly) in S–waves. Therefore the quantum numbers of the constituents already define the molecules they can (most easily) form. In addition, only narrow states can form hadronic molecules, since a shallow bound state that contains a broad building block would be very short lived or might not even have the time to be formed before the constituent decays [9, 10].

In addition, it appears natural to expect that the one–pion exchange plays a crucial role in the formation of the bound states — after all it is also understood as a crucial ingredient to the nuclear force. In this context it is important to acknowledge that the strength of the one–pion exchange changes by a factor -1/3 when switching from an isoscalar to an isovector channel. In addition, it also changes sign, when systems of opposite C–parity are looked at. As a result of this one should expect that, if there is an isoscalar molecule of a given C–parity, the isovector partner, if it exists, should have opposite C–parity [5]. This is in contrast to the tetraquark picture where for each quantum number there should always be both an isoscalar and an isovector state.

Facing the Experiment

The quantum numbers of the $X(3872)$, discovered in $B^\pm \to K^\pm X$ ($X \to J/\psi\pi^+\pi^-$) by BELLE [11], soon after confirmed by BABAR [12], have been determined by LHCb to be $J^{PC} = 1^{++}$ [13] (see also the more refined analysis presented in Ref. [14]). Especially since the mass of $X(3872)$ is located 100 MeV lower than that predicted for the $\chi_{c1}(2P)$ [15] and since it decays equally probable to into the isovector channel $\rho J/\psi$ and the isoscalar channel $\omega J/\psi$ the $X(3872)$ was regarded as exotic candidate. The mass of the $X(3872)$ lies extremely close to the $D^0\bar{D}^{*0}$ threshold and therefore the most natural explanation for this state might be a 1^{++} $D\bar{D}^*$ molecule [16]. As a consequence of the separation to the D^+D^{*-} channel of only 8 MeV, strong isospin breaking is predicted in this szenario [16, 17]. Indeed, the comparable rates in the $\omega J/\psi$ and $\rho^0 J/\psi$ channels are consistent with an interpretation of $X(3872)$ as an isoscalar $D\bar{D}^*$ molecule when the different widths of ρ and ω as well as the mass difference between the $D\bar{D}^*$ thresholds are taken into account [18]. A four-quark state $cq\bar{c}\bar{q}'$ was also discussed [19, 20]. However, since the charged partners of the $X(3872)$ have not been observed (e.g. in $B^- \to \bar{K}^0 X^-$ nor $B^0 \to K^+ X^-$, where $X^- \to J/\psi\,\pi^-\pi^0$ [21] this explanation appears unlikely. The claim that $X(3872)$ must be a compact (tetraquark) state, since it is also produced at very high p_T in $\bar{p}p$ collisions [22], was challenged in [23] which stresses the importance of rescattering, see also [24].

The $Y(4260)$ was discovered in $J/\psi\pi^+\pi^-$ by BABAR in initial state radiation [25]. A charmonium state with the quantum numbers 1^{--} is not expected in this mass region. In addition, if $Y(4260)$ were a charmonium, it should have a significant coupling to $\bar{D}D$. However, this decay channel is not observed. This state could be a hybrid charmonium with a spin-1 $\bar{c}c$ core [26, 27]. However, provided that the observation of $Y(4260) \to h_c(1P)\pi\pi$ by BESIII [28] is confirmed, the hybrid hypothesis would be under pressure, since this transition would violate spin symmetry.

The same criticism applies to the hadrocharmonium interpretation of the $Y(4260)$ which describes this state as spin-1 quarkonium surrounded by a light quark cloud [3]. To circumvent the spin symmetry argument [4] argues that $Y(4260)$ and $Y(4360)$ could be mixtures of two hadrocharmonia with spin triplet and spin singlet heavy quark pairs. The same kind of mixing could also operate for a hybrid.

The mixing scenario of Ref. [4] opens an interesting opportunity: using the proposed scenario for $Y(4260)$ and $Y(4360)$ as input one can use spin symmetry to predict in total 4 spin partners of the mentioned states — most special amongst them is a pseudo–scalar state, predicted to decay predominantly into $\eta_c^{(\prime)}\pi\pi$, which is significantly lighter than $Y(4260)$ [5].

In Ref. [29] the $Y(4260)$ was conjectured to be predominantly a $D_1\bar{D}$ molecule. For the authors this assignment not only provided a natural mechanism for the production of a $D\bar{D}^*$ molecule, $Z_c(3900)$, but also allowed subsequently for the prediction of the copious production of $X(3872)$, also assumed to be a $D\bar{D}^*$ molecular state, in $Y(4260)$ radiative decays [30]. This prediction was confirmed shortly after at Belle [31]. The $Y(4360)$ as $D_1\bar{D}^*$ bound state could be the spin partner of the $Y(4260)$ [32, 33], but a detailed microscopic calculation to make this connection solid is still lacking.

The tetraquark picture of Maiani et al. explains the observed Y states and, when including a tailor-made spin-spin interaction, is also capable to describe both $Z_c(3900)^{\pm}$ and $Z_c(4020)^{\pm}$ [34], and even the recently confirmed $Z(4430)^{\pm}$. However, even when counting only the ground states, the model predicts 80 states in the mass range 3.7 - 4.36 GeV, with at least 67 still to be found [5]. In particular, each tetraquark model predicts for every isoscalar state an isovector partner, neither of which found up today although intensive searches were performed.

The charged states $Z_c(3900)^{\pm}$, first observed by BESIII [35] and the to be confirmed $Z_c(4020)^{\pm}$ [36] decay predominantly into $\bar{D}D^*$ and $\bar{D}^*D^*$, respectively, while $Z_b(10610)^{\pm}$ and $Z_b(10650)^{\pm}$ [37, 38] decay predominantly into $\bar{B}B^*$ and $\bar{B}^*B^*$, respectively, although all of them were discovered in decay modes with a heavy quarkonium and a pion. This suggests that the states are close relatives and their interactions connected via heavy quark flavor symmetry. A molecular interpretation for the bottomonium states was proposed shortly after the discovery of the $Z_b^{\pm}$ states [39] and also shortly after that of the $Z_c(3900)^{\pm}$ [29]. However, their properties also appear to be consistent with tetraquark structures [40].

The heaviest charged state in the charmonium sector is the $Z(4430)^{\pm}$ observed by BELLE [41] and confirmed at LHCb [42] It is interpreted as hadrocharmonium [3], $\bar{D}_1D^*$ molecule [43] as well as tetraquark [34]. Ref. [44] provided an explanation of this state as cross channel effect, without introducing an explicit pole.

LATTICE QCD

Lattice methodology

Lattice QCD is a reliable non-perturbative method to study hadron properties based directly on QCD. It relies on numerical path integration in Eucledian discretized and finite space-time. The physics information on a hadron (below, near or above threshold) is obtained from the discrete energy spectrum E_n, which is extracted from the time-dependence of the correlation functions $\langle\Omega|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|\Omega\rangle = \sum_n Z_i^n Z_j^{n*}\ e^{-E_n t}$, where $Z_i^n \equiv \langle\Omega|\mathcal{O}_i|n\rangle$. The $\mathcal{O}_i$ are the interpolating fields that create/annihilate the physical system with given quantum numbers J^{PC}: for quarkonium(like) states one takes for example $\mathcal{O} \simeq \bar{Q}\Gamma Q$, two-meson interpolators $\mathcal{O} = (\bar{Q}\Gamma_1 q)(\bar{q}\Gamma_2 Q)$, $(\bar{Q}\Gamma_1 Q)(\bar{q}\Gamma_2 q)$ and $\mathcal{O} = [\bar{Q}\Gamma_1\bar{Q}][c\Gamma_2 q]$ with $Q = c, b$ and $q = u, d, s$. The energy eigenstates $|n\rangle$ are predominantly "one-meson" states (e.g. χ_{c0}) or predominantly "two-meson" states (e.g. $D\bar{D}$) - in interacting theory they are mixtures of those.

In the energy region near or above threshold, the masses of bound-states and resonances have to be inferred from the infinite-volume scattering matrix $S(E)$ of the one-channel (elastic) or multiple-channel (inelastic) scattering of two-hadrons. This has been done for QCD in practice only during few recent years and two approaches are briefly summarized here. The poles of the resulting $S(E)$ then provide the masses of resonances and bound states. Lüscher has shown that the energy E of two-meson eigenstate in finite volume gives the scattering matrix $S(E)$ at that energy in infinite volume [45]. This leads to $S_l(E) = e^{2i\delta_l(E)}$ for partial wave l only for specific values of E since the spectrum of two-meson eigenstates on finite volume is discrete. The other approach to extract $S(E)$ was proposed by HALQCD [46], which starts by determining the two-hadron Bethe-Salpeter wave function and two-hadron potential from the lattice. The phase shift $\delta(E)$ and $S(E)$ are then obtained using the Schrödinger equation for given $V(r)$. The Lüscher approach has been thoroughly verified on the conventional resonances like ρ, while HALQCD approach has not been verified on resonances yet. There are ongoing discussions as to whether HALQCD approach is as rigorous as the Lüscher-type approach.

All presented simulations of heavy quarkonia omit Wick contractions where $\bar{Q}Q$ pair is annihilated, while all other Wick contractions are taken into account.

Spectrum of quarkonia below open-flavor threshold

The masses $m_i = E_i(P=0)$ of charmonia well below $D\bar{D}$ threshold are extracted from correlation functions obtained with $\bar{c}c$ interpolating fields, which are extrapolated to continuum, infinite volume and physical quark masses. Recent precision spectra [49–52] are in impressive agreement with experimental masses, and there are no major open issues. The main remaining uncertainty is due to the omission of $\bar{c}c$ annihilation.

The spectrum of bottomonia below $B\bar{B}$ contains many more states. The recent lattice spectrum [47] in Fig. 3 presents a valuable reference point where to expect states that have not been found in experiment yet. A possible bottomonium analog of $X(3872)$ near $\bar{B}B^*$ threshold has not been found on the lattice or in the experiment (yet).

Excited charmonia within single-meson approach

The most extensive spectra of the excited charmonia have been calculated within the so-called single-meson approach Hadron Spectrum Collaboration in 2012 [48]. Several complete quark-antiquark multiplets nL were found. Multiplets of hybrid states were also found and some of them carry exotic J^{PC}. The single-meson treatment ignores strong decays of resonances and threshold effects. It gives valuable reference spectra, but can not give reliable conclusions on the existence of the near-threshold exotic states, for example.

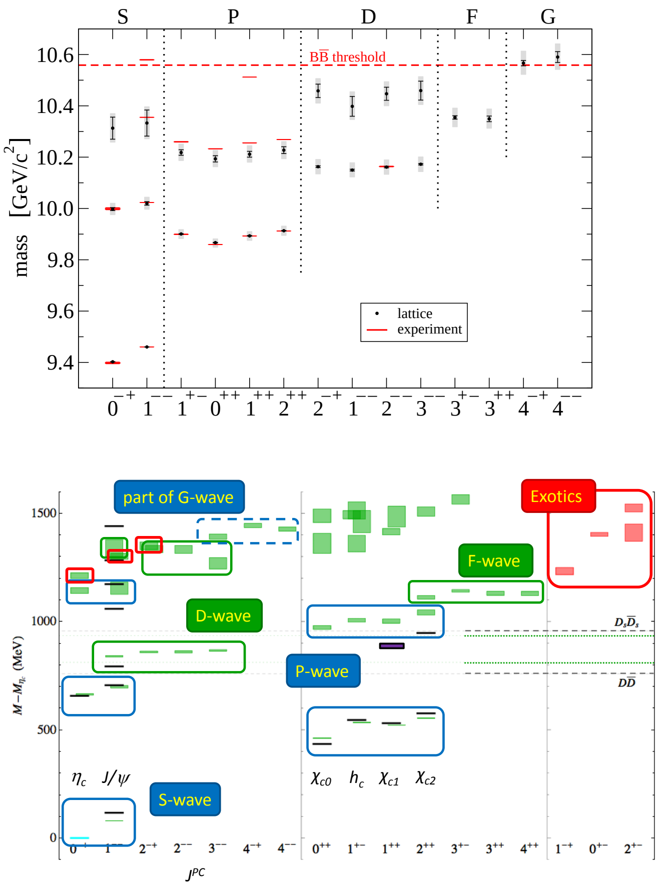

FIG. 3: Up: Lattice spectra of $\bar{b}b$ from [47]. Bottom: Lattice spectra of $\bar{c}c$ within single-meson approximation from [48] are given by green lines, while experimental states are shown by black lines. Hybrid candidates are encircled by red, while other nL multiplets are encircled by blue or green.

Vector and scalar resonances

Until recently, all quarkonia above $D\bar{D}$ ($B\bar{B}$) threshold were treated ignoring their strong decay to a pair of charmed (beauty) mesons. The first exploratory simulation aimed at determining the masses as well as the decay widths of these resonances using Lüscher's approach was presented in [53]. The Breit-Wigner-type fit of the $D\bar{D}$ scattering matrix $S(E)$ for the p-wave leads to the resonance mass and width [**?**] of $\psi(3770)$, which agree with experiment within errors (see Fig. 4). The $\psi(2S)$ in Fig. 4 appears as a bound state pole below threshold.

In the scalar channel, only the ground state $\chi_{c0}(1P)$ is understood and there is no commonly accepted candidate for its first excitation. PDG assigns it to $X(3915)$, but this is seriously questioned in [57–59]. The scattering matrix for $D\bar{D}$ in s-wave was extracted from lattice [53], but this also does not allow a clear answer to the puzzles. The lattice data provide an indication for a yet-

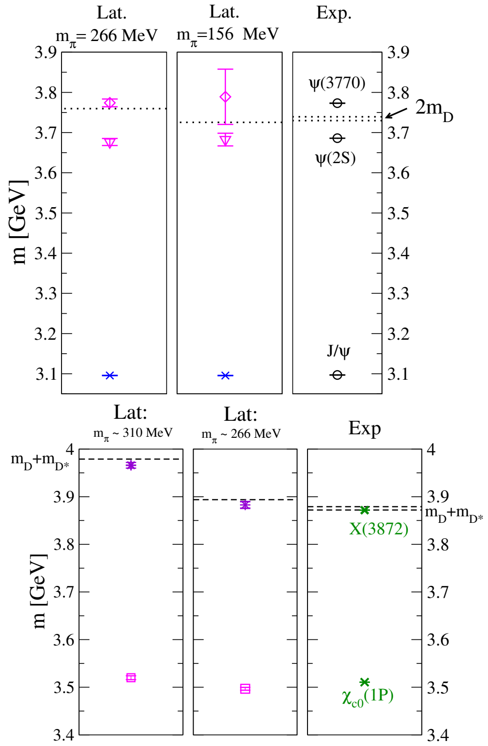

FIG. 4: Up: The spectrum of the vector charmonia from [53]: the diamond denotes the resonance mass of $\psi(3770)$, while the triangle denotes the pole mass of the bound state $\psi(2S)$; both are obtained from $D\bar{D}$ scattering matrix. Bottom: The location of $X(3872)$ with $I = 0$ which emerges as shallow bound state in $D\bar{D}^*$ scattering at $m_\pi \simeq 266$ MeV [54] and $m_\pi \simeq 310$ MeV [55] (update of [56]).

unobserved narrow resonance slightly below 4 GeV with $\Gamma[\chi'_{c0} \to D\bar{D}]$ below 100 MeV. Among scenarios considered, only a scenario with this narrow resonance and a pole in the $D\bar{D}$ scattering matrix at $\chi_{c0}(1P)$ agrees with the energy-dependence of the scattering matrix [53]. Further experimental and lattice QCD efforts are required to map out the $D\bar{D}$ and $J/\psi\omega$ scattering in more detail.

Charmonium-like states $X(3872)$ and $Y(4140)$

The $X(3872)$ lies experimentally on $D^0\bar{D}^{0*}$ threshold and its existence on the lattice can not be established without taking into account the effect of this threshold. This was first done by simulating $D\bar{D}^*$ scattering in [54], where $D\bar{D}^*$ scattering matrix in $I(J^{PC}) = 0(1^{++})$ channel was determined using Lüscher's approach. The pole was found just below the threshold and it was associated with a bound state $X(3872)$; its location is compared to the experimental mass in Fig. 4. The more recent simulation using HISQ action confirms the existence of the pole just below the threshold [55, 56].

The lattice study [60] investigated which Fock components are essential for appearance of $X(3872)$ with $I = 0$ on the lattice. The energy eigenstate related to $X(3872)$ appears in the simulation only if $D\bar{D}^*$ as well as $\bar{c}c$ interpolating fields are employed. The $X(3872)$ does not appear in absence of $\bar{c}c$ interpolators, even if (localized) interpolators $[\bar{c}\bar{q}]_{3_c}[cq]_{\bar{3}_c}$ or $[\bar{c}\bar{q}]_{6_c}[cq]_{\bar{6}_c}$ are in the interpolator basis. This indicates that $\bar{c}c$ Fock component is most likely more essential for $X(3872)$ than the diquark-antidiquark one.

The search for charged $X(3872)$ with $J^{PC} = 1^{++}$ was performed in [60]. Only expected two-meson eigenstates were found and no candidate for charged X. The reliable search for the neutral $I = 1$ state would need to incorporate isospin breaking effects [61], but that has not been performed on the lattice yet.

The experimental candidate $Y(4140)$ with hidden strangeness was experimentally observed in $J/\psi\phi$ invariant mass and has unknown J^P at present. The lattice search for it was performed in $J^{PC} = 1^{++}$ channel with $\bar{c}c\bar{s}s$ interpolators and no candidate was found [60]. The s-wave and p-wave $J/\psi\ \phi$ scattering matrix from [62], which omits $\bar{s}s$ annihilation, do not support the resonant structure.

Charged quarkonium-like states $Z^+_{c,b}$

The lattice search for the manifestly exotic states Z_c^+ with flavour content $\bar{c}c\bar{d}u$ and $I^G(J^{PC}) = 1^+(1^{+-})$ is very challenging since the experimental candidates lie above several thresholds and can decay in several final states via strong interaction. The reliable treatment requires the extraction of coupled channel scattering matrix. This has been done using the Lüschers method only for one system in the light sector [66].

The search for Z_c^+ has therefore been performed using a simplified approach in [67]. The challenge is that there are many two-meson eigenstates ($J/\psi\pi$, $D\bar{D}^*$, ...) in the relevant energy region. The simulation of all these coupled channels (including diquark-antidiquark interpolators) renders all expected two-meson eigenstates, but no additional energy eigenstate, therefore no candidate for Z_c^+. This has been recently confirmed by a simulation [56]. Note that an extra eigenstate (in addition to expected two-meson states) has been found for all resonances and all bound states that have been established on the lattice up to now. The absence of an additional eigenstate for $Z_c(3900)$ could indicate that this experimental state is not related to a conventional resonance pole, but could be due to the coupled channel effect.

This possibility was investigated within the HALQCD approach [46] to extract the scattering matrix of the coupled channels $D\bar{D}^*$, $J/\psi\pi$ and $\eta_c\rho$ [63]. The 3×3 matrix of two-hadron potentials $V(r)$ in Fig. 5 shows that off-diagonal potential between channels $\pi J/\psi$ and $D\bar{D}^*$ is larger than other potentials. The potentials render

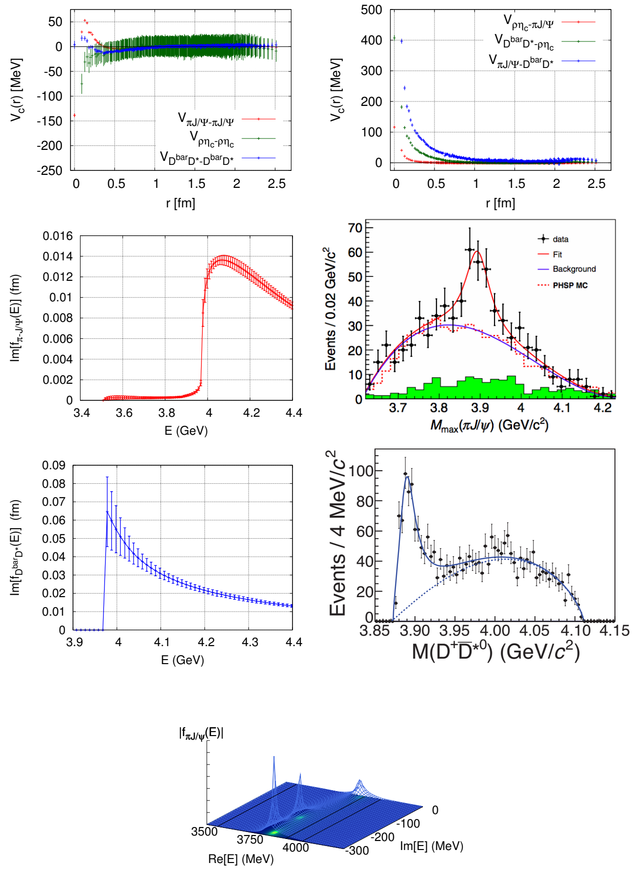

FIG. 5: The HALQCD results related to $Z_c^+(3900)$ where scattering amplitude f is defined through $d\sigma/d\Omega = |f(E)|^2$ [63]. Top left: potentials $V(r)$ for three channels. Top right: crossed-channel potentials $V(r)$ with sizable $J/\psi\pi - D\bar{D}^*$. Second and third row: the number of particles $Im[f_i] \propto j_i\sigma_i$ for channels $i = J/\psi\pi$, $D\bar{D}^*$ compared to the experimental shapes for $J/\psi\pi$ [64] and $D\bar{D}^*$ [65]. Bottom: Poles of the scattering matrix in the complex energy plane.

3×3 scattering matrix [68]. This is turn leads to the number of events $N_{\pi J/\psi}$ and $N_{D\bar{D}^*}$ ($N_i \propto \sigma_i j_i$) with enhanced peaks above $D\bar{D}^*$ threshold that resemble experimental line shapes for $Z_c^+(3900)$. Two poles of the scattering matrix in the complex energy plane are found at $Im[E_{cm}]<0$ and $Re[E_{cm}]<m_D+m_{\bar{D}^*}$. The authors conclude that $Z_c^+(3900)$ most likely does not correspond to the conventional resonant pole, while the cross section peak is related to sizeable $\pi J/\psi - D\bar{D}^*$ coupled channel effect. These conclusions need to be verified using the Lüscher-type approach.

The scattering matrices for s-wave and p-wave near $D\bar{D}^*$ threshold were determined using only $D\bar{D}^*$ interpolating fields in [69], which may not be reliable since the ground state of the system is $J/\psi\pi$. The authors conclude that no evidence for $Z_c^+(3900)$ is found.

The lattice search for a pair of Z_b^+ from Belle [70] has not been performed yet, as each of them can decay to at least 5 two-meson final states, which makes the problem even more challenging.

Pentaquarks

The NPLQCD collaboration finds an interesting indication for a $\eta_c N$ bound state approximately 20 MeV below $\eta_c N$ threshold [71] (N denotes nucleon). This is the only pentaquark candidate containing $\bar{c}c$ from lattice studies up to now. As the simulation is done at $SU(3)$ flavour symmetric point corresponding to $m_\pi \simeq$ 800 MeV, it is not clear yet whether this bound state persists to physical m_π.

The lattice simulation of two pentaquarks P_c discovered by LHCb [72] will be much more challenging as they are located 0.4 GeV above $J/\psi p$ threshold and they have several open decay channels. Such lattice results are not available yet.

$\bar{Q}\bar{Q}qq$ tetraquarks

The $\bar{Q}\bar{Q}qq$ bound states were searched by determining the potentials of two static antiquarks Q in the presence of two finite mass quarks q, followed by solving the Schrodinger's equation. The indication for a bound state with flavor $ud\bar{b}\bar{b}$ and quantum numbers $I(J^P)=0(1^+)$ was found in [73–75]. The channels $\bar{b}\bar{b}qq$ with $I=1$, $\bar{b}\bar{b}cc$, $\bar{b}\bar{b}ss$ and $J^P=0^+,1^+,2^+$ do not contain bound states at $m_\pi = 340$ MeV [74].

Tetraquarks containing charm are simulated with non-static charm. The s-wave potentials in channels DD, $\bar{K}D$, DD^* and $\bar{K}D^*$ interactions with $cc\bar{u}\bar{d}$ and $cs\bar{u}\bar{d}$ were determined with HALQCD method [76]. Repulsive interaction is obtained for $I=1$ channels and an attractive interaction for $I=0$ channels. No bound states or resonances are observed at simulated $m_\pi \geq 410$ MeV; the attraction becomes more prominent at lighter m_π and that there is some indication that BB^* with $I(J^P)=0(1^+)$ could be bound [76]. The discrete energies of eigenstates for the $qq\bar{c}\bar{c}$ system with $q=u,d$ were presented in a preliminary study [77]. Expected two-meson eigenstates were found, but no extra eigenstate that could be related to the exotics.

Radiative transitions and leptonic widths of quarkonia

Pending (works of HPQCD, Lewis et al., Becirevic et al,...)

Outlook

Spectra of charmonia and bottomonia below open-flavor threshold are precise, under control and in reasonable agreement with experiment. The decay constants and radiative transitions between some of these states have also been determined. The matrix elements that concern the excited states (still below threshold) are tractable in the near future. The main unresolved uncertainty comes from the neglect of $\bar{Q}Q$ annihilation, which invokes decays to light hadrons and presents a huge challenge.

Information on the states above or slightly below threshold have to be inferred from the scattering matrix extracted on the lattice. The hadronic resonances that reside in a single scattering channel can be treated rigorously by simulating the scattering in this channel. Same applies for hadrons that are situated slightly below the corresponding threshold. Reasonable results can be obtained also for the hadrons where a coupling to one channel is dominant, while coupling to others may be neglected. For example, the first results on $\psi(3770)$, $X(3872)$ and $\chi_{c0}(2P)$ in $D\bar{D}^{(*)}$ have been obtained by simulating the $D\bar{D}^{(*)}$ scattering, and the results with improved accuracy can be expected soon.

The states that can decay hadronically into two different two-meson final states are challenging, but manageable. The scattering matrix for the two-coupled channels has been recently extracted for the first time using the Lüscher-tye method [66], so the analogous results relevant to quarkonium(like) spectroscopy at Belle 2 can be expected by the time it starts operating. The preliminary results on the three-coupled channels from HALQCD method indicate that the experimental $Z_c(3900)$ peak may be related to the large coupling between $J/\psi\pi$ and $D\bar{D}^*$ channels.

Belle 2 will aim also the quarkonium and quarkonium-like states that are located above multiple thresholds, for example Z_b and $Z^+(4430)$. Every additional two-hadron state with given J^{PC} that appears below the state of interest represents significant increase of effort and challenge for lattice QCD. Some of these states will have to be studied under certain simplified assumptions. A significant amount of analytical work and numerical studies will be needed to understand to what extent these assumptions could be reliable, and how to overcome them.

PROCESSES

B-decays

Initial–State–Radiation

Two–Photon Collisions

Double–Charmonium Production

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